FOR CLASS XII 20-21 – MATHEMTICS – TEACHER MANISHA ROY

TOPIC – INVERSE TRIGONOMETRIC FUNCTIONS ASSIGNMENT

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CLASS-12 **IMPORTANT POINTS** MATHEMATICS

1. **Definitions of inverse trigonmoetric functions** chapter-2

I. The function sin x is one-one and onto from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ to [-1, 1] and its inverse function $\sin^{-1}$ has domain [-1, 1] and range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

II. The function $\cos x$ is one-one and onto from [0, π] to [-1, 1] and its inverse function $\cos^{-1} x$ has domain [-1, 1] and range [0, π].

III. The function $\tan x$ is one-one and onto from $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\tan^{-1} x$ has domain R and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

IV. The function $\cot x$ is one-one and onto from (0, π), to R and its inverse function $\cot^{-1} x$ has domain R and range (0, π).

V. The function $\sec x$ is one-one and onto from [0, π] - {π/2} to R - (-1, 1) and its inverse function $\sec^{-1} x$ has domain R - (-1, 1) and range [0, π] - {π/2}.

VI. The function cosec x is one-one and onto from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ - {0} to R - (-1, 1) and its inverse function $cosec^{-1}x$ has domain R - (-1, 1) and range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ - {0}.

2. **Values of inverse trigonometric functions**

I. For $x \in$ [-1, 1], the value of $\sin^{-1} x = \theta$ if $\sin\theta = x$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

II. For $x \in$ [-1, 1], the value of $\cos^{-1} x = \theta$ if $\cos\theta = x$ and $\theta \in$ [0, π]

III. For $x \in$ R, the value of $\tan^{-1} x = \theta$ if $\tan\theta = x$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

IV. For $x \in$ R, The value of $\cot^{-1} x = \theta$ if $\cot\theta = x$ and $\theta \in$ (0, π).

V. For $x \in$ R - (-1, 1), the value of $\sec^{-1} x = \theta$ if $\sec\theta = x$ and $\theta \in$ [0, π] - {π/2}.

VI. For $x \in R$ - (-1, 1), the value of $\text{cosec}^{-1}x = \theta$ if $\text{cosec}\theta = x$ and $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ - {0}.

VII. If is positive or zero then the values of all inverse trigonometric functions lie between 0 and $\pi/2$ inclusive. If is negative, then the values of lie between- $\pi/2$ and 0 inclusive and the values of lie between $\pi/2$ and inclusive.

3. **Important formulae**

I. (i) $\sin^{-1}(\sin\theta) = \theta,\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(ii) $\cos^{-1}(\cos\theta) = \theta,\ \theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta,\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(iv) $\cot^{-1}(\cot\theta) = \theta,\ \theta \in (0, \pi)$

(v) $\sec^{-1}(\sec\theta) = \theta,\ \theta \in [0, \pi] - \{\pi/2\}$

(vi) $\text{cosec}^{-1}(\text{cosec}\theta) = \theta,\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

II. (i) $\sin(\sin^{-1}x) = x,\ x \in [-1, 1]$

(ii) $\cos(\cos^{-1}x) = x,\ x \in [-1, 1]$

(iii) $\tan(\tan^{-1}x) = x,\ x \in R$

(iv) $\cot(\cot^{-1}x) = x,\ x \in R$

(v) $\sec(\sec^{-1}x) = x,\ x \in R - (-1, 1)$

(vi) $\text{cosec}(\text{cosec}^{-1}x) = x,\ x \in R - (-1, 1)$

III. (i) $\sin^{-1}(-x) = -\sin^{-1}x,\ x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x,\ x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x,\ x \in R$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x,\ x \in R$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x,\ x \in R - (-1, 1)$

(vi) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x,\ x \in R - (-1, 1)$

IV. (i) $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x \quad \text{for } x \le -1 \quad or \quad x \ge 1$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x \quad \text{for } x \le -1 \quad or \quad x \ge 1$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x & \text{for} \quad x > 0 \\ -\pi + \cot^{-1} x & \text{for} \quad x < 0 \end{cases}$

V. (i) $\sin^{-1} x + \cos^{-1} x = \pi/2,\ x \in [-1,\ 1]$

(ii) $\tan^{-1} x + \cot^{-1} x = \pi/2,\ x \in R$

(iii) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \pi/2,\ x \in R - (-1,\ 1)$

VI. (i) (a) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy},\ \text{if } xy < 1$

(b) $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy} \quad \text{if } x > 0,\ y > 0,\ xy > 1$

(c) $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1} \frac{x+y}{1-xy} \quad \text{if } x < 0,\ y < 0,\ xy > 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy},\ \text{if } xy > -1$

VII. (i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

if either $x^2 + y^2 \le 1$ *or* $xy < 0,\ |x| \le 1,\ |y| \le 1$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

if either $x^2 + y^2 \le 1$ *or* $xy > 0,\ |x| \le 1,\ |y| \le 1$

VIII. (i) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right] \text{if } x + y \ge 0,\ |x| \le 1,\ |y| \le 1$

(ii) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right] \text{if } x \le y,\ |x| \le 1,\ |y| \le 1$

IX. (i) $\sin^{-1} \frac{2x}{1+x^2} = 2\tan^{-1} x,\ x \in [-1,\ 1]$

(ii) $\cos^{-1} \frac{1-x^2}{1+x^2} = 2\tan^{-1} x,\ x \in [0,\ \infty)$

(iii) $\tan^{-1} \frac{2x}{1-x^2} = 2\tan^{-1} x,\ x \in (-1,\ 1)$

X. (i) $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right),\ x \in \left[-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]$

(ii) $2\cos^{-1} x = \cos^{-1}\left(2x^2 - 1\right),\ x \in [0,\ 1]$

INVERSE TRIGONOMETRIC FUNCTIONS

(1 Mark Questions)

1. Find the principal values of the following:

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\cos^{-1}\left(\frac{-1}{2}\right)$

(iii) $\cot^{-1}\left(-\sqrt{3}\right)$

(iv) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(v) $\sec^{-1}\left(-\sqrt{2}\right)$

(vi) $\sin^{-1}\sin\left(\frac{4\pi}{5}\right)$

(vii) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(viii) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$

(ix) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$

(x) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

ANSWERS

1. (i) $\frac{\pi}{3}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{5\pi}{6}$

(iv) $\frac{\pi}{6}$ (v) $\frac{3\pi}{4}$ (vi) $\frac{\pi}{5}$

(vii) $\frac{5\pi}{6}$ (viii) $\frac{-\pi}{6}$ (ix) $\frac{-\pi}{3}$

(x) $\frac{\pi}{3}$

(4 Marks Questions)

1. Find the values of the following:

(i) $\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(\frac{1}{2}\right)$ (ii) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

(iii) $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ (iv) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

(v) $\sin^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{4}{5}\right)$ (vi) $\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(vii) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (viii) $\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

2. Solve for x

(i) $\tan^{-1}x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

3. Simplify the following:

(i) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (ii) $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

(iii) $\tan^{-1}\left(\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right)$ (iv) $\cot^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

4. Prove that

(i) $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \frac{1}{2}\cos^{-1}(3/5)$

(ii) $\sin^{-1}(4/5) + 2\tan^{-1}(1/3) = \pi/2$

(iii) $\sin^{-1}(5/13) + \cos^{-1}(3/5) = \tan^{-1}(63/16)$.

Inverse

1. $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)=\frac{17}{6}$

2. $\cos^{-1}(3/5)+\cos^{-1}(12/13)+\cos^{-1}(63/65)=\pi/2$

3. $4\left(\sin^{-1}\frac{1}{\sqrt{10}}+\cos^{-1}\frac{2}{\sqrt{5}}\right)=\pi$

4. Simplify $\cot^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

5. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi/2$, prove that $yz+zx+xy=1$

6. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, prove that $x+y+z=xyz$

7. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$

8. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-2\frac{xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$

9. Solve the following equations.

(i) $\tan^{-1}\left(\frac{x-1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{2x+1}\right)=\tan^{-1}(23/36)$

(ii) $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$

(iii) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

(iv) $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

10. Simplify :

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right), x\neq 0$

(ii) $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right), 0<x<1$

(iii) $\tan^{-1}\left(\frac{x}{a+\sqrt{a^2-x^2}}\right), -a < x < a$

11. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, prove that $x=\frac{a+b}{1-ab}$

12. Prove that :

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=\frac{2b}{a}$$

13. Prove that :

(i) $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]=\frac{1}{2}\sin^{-1}\left(x^2\right), |x|<1$

(ii) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(x^2\right), |x|<1$

14. Simplify :

(i) $\sin^{-1}\left[\frac{\sin x+\cos x}{\sqrt{2}}\right], -\frac{\pi}{4}<x<\frac{\pi}{4}$

(ii) $\cos^{-1}\left[\frac{\sin x+\cos x}{\sqrt{2}}\right], \frac{\pi}{4}<x<\frac{5\pi}{4}$

15. Prove that :

(i) $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{\pi}{2}-\frac{x}{2}$, if $\frac{\pi}{2}<x<\pi$

(ii) $\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right]=\frac{\pi}{4}-\frac{x}{2}$, if $\pi<x<\frac{3\pi}{2}$

ANSWERS (4 Marks)

1. (i) $\frac{2\pi}{3}$ (ii) π (iii) 1

(iv) $\frac{\pi}{4}$ (v) $\sin^{-1}\left(\frac{6+4\sqrt{5}}{15}\right)$ (vi) $\frac{3\pi}{2}$

(vii) $\frac{-\pi}{6}$ (viii) $\frac{\pi}{2}$

2. (i) 1 (ii) $\frac{\sqrt{3}}{2}$

3. (i) $\frac{x}{2}$ (ii) $\frac{x}{2}$ (iii) $\frac{\pi}{4}+\frac{x}{2}$

(iv) $\frac{\pi}{4}-\frac{x}{2}$

9. (i) $\frac{4}{3}$ (ii) 0, ± 1 (iii) $\frac{1}{4}$ (iv) 0, ± 1/2

10. (i) $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}x$ (ii) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$ (iii) $\frac{1}{2}\sin^{-1}\frac{x}{a}$

14. (i) $x+\frac{\pi}{4}$ (ii) $x-\frac{\pi}{4}$